AP Calculus – Final Review Sheet

When you see the words ….	This is what you think of doing
1. Find the zeros	Set function = 0 Factor / Quad. Formula / Calculator
2. Find equation of the line tangent to $f(x)$ at (a,b)	Point: (a, b) Slope: $f'(a)$ $y - y_1 = f'(a)(x - x_1)$
3. Find equation of the line normal to $f(x)$ at (a,b)	Point: (a, b) Slope: $-\frac{1}{f'(a)}$ $y - y_1 = -\frac{1}{f'(a)}(x - x_1)$
4. Show that $f(x)$ is even	$f(-x) = f(x)$ y-axis symmetry
5. Show that $f(x)$ is odd	$f(-x) = -f(x)$ origin symmetry
6. Find the interval where $f(x)$ is increasing	Set $f'(x) = 0$ or $f'(x) =$ undefined (Critical Values Determine where $f'(x) > 0$
7. Find interval where the slope of $f(x)$ is increasing	$f'(x)$ is inc. when $f''(x) > 0$ Set $f''(x) = 0$ or $f''(x) =$ undefined Determine where $f''(x) > 0$
8. Find the minimum value of a function	Make sign chart for $f'(x)$. Find x where $f'(x)$ changes from $-$ to $+$. Plug x's into $f(x)$ and choose smallest.
9. Find the minimum slope of a function	Make sign chart for $f''(x)$. Find x where $f'(x)$ changes from $-$ to $+$. Plug x's into $f(x)$ and choose smallest.
10. Find critical values	Find $f'(x) = 0$ and $f'(x) =$ undefined
11. Find inflection points	Find $f''(x) = 0$ and $f''(x) =$ undefined Make sign chart for $f''(x)$ and see where $f''(x)$ changes sign $(+ \text{ to } -)$ or $(- \text{ to } +)$.
12. Show that $\lim_{x \to a} f(x)$ exists	Show that $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
13. Show that $f(x)$ is continuous	1) $\lim_{x \to a} f(x)$ exists 2) $f(a)$ exists 3) $\lim_{x \to a} f(x) = f(a)$
14. Find vertical asymptotes of $f(x)$	Factor and cancel $f(x)$ Set denominator = 0
15. Find horizontal asymptotes of $f(x)$	Find: $\lim_{x \to \infty} f(x)$ $\lim_{x \to -\infty} f(x)$

16. Find the average rate of change of $f(x)$ on $[a,b]$	$\frac{f(b)-f(a)}{b-a}$
17. Find instantaneous rate of change of $f(x)$ at a	$f'(a)$
18. Find the average value of $f(x)$ on $[a,b]$	$\frac{1}{b-a}\int_a^b f(x)\,dx$
19. Find the absolute maximum of $f(x)$ on $[a,b]$	Make sign chart for $f'(x)$. Find x's where $f'(x)$ changes from + to −, and check endpoints. Plug x's into $f(x)$ and choose largest.
20. Show that a piecewise function is differentiable at the point a where the function rule splits	Make sure function is continuous at $x=a$. Show $\lim_{x\to a^-} f'(x) = \lim_{x\to a^+} f'(x)$
21. Given $s(t)$ (position function), find $v(t)$	$s'(t) = v(t)$
22. Given $v(t)$, find how far a particle travels on $[t_1,t_2]$	$\int_a^b \lvert v(t)\rvert\,dt$
23. Find the average velocity of a particle on $[t_1,t_2]$	$\frac{1}{b-a}\int_a^b v(t)\,dt = \frac{s(b)-s(a)}{b-a}$
24. Given $v(t)$, determine if a particle is speeding up at $t=k$	Speeding up if signs of $v(k)$ and $a(k)$ are same slowing down if signs of $v(k)$ and $a(k)$ are different
25. Given $v(t)$ and $s(0)$, find $s(t)$	$s(t) = \int v(t)\,dt + s(0)$
26. Show that Rolle's Theorem holds on $[a,b]$	Show that f is continuous and differentiable. If $f(a)=f(b)$, then find c in $[a,b]$ such that $f'(c)=0$.
27. Show that Mean Value Theorem holds on $[a,b]$	Show that f is continuous and differentiable Find c: $f'(c) = \frac{f(b)-f(a)}{b-a}$
28. Find domain of $f(x)$	Domain Restrictions: 1) Non-Zero Denominators 2) Square Root of non-negative #'s 3) log or ln of positive #'s
29. Find range of $f(x)$ on $[a,b]$	Find relative max/mins Examine y-values of max/mins, and endpoints.
30. Find range of $f(x)$ on $(-\infty,\infty)$	Find relative max/mins Examine y-values of max/mins and $\lim_{x\to\pm\infty} f(x)$
31. Find $f'(x)$ by definition	$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ OR $f'(x) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

32. Find derivative of inverse to $f(x)$ at $x = a$	$(f^{-1})' = \frac{1}{f'}$
33. y is increasing proportionally to y	$\frac{dy}{dt} = ky \longleftrightarrow y = y_0 e^{kt}$
34. Find the line $x = c$ that divides the area under $f(x)$ on $[a,b]$ to two equal areas	$\int_a^c f(x)\,dx = \int_c^b f(x)\,dx$ OR $\int_a^c f(x)\,dx = \frac{1}{2} \cdot$ Total Area
35. $\frac{d}{dx}\int_a^x f(t)\,dt =$	$f(x)$
36. $\frac{d}{dx}\int_a^u f(t)\,dt$	$f(u) \cdot \frac{du}{dx}$
37. The rate of change of population is ...	$\frac{dP}{dt} = \ldots$
38. The line $y = mx + b$ is tangent to $f(x)$ at (a,b)	1) The two functions share the same slope ($m = f'(x)$) 2) The two functions share the same point (a,b)
39. Find area using left Riemann sums	$A = \Delta x\,[y_0 + y_1 + y_2 + \ldots + y_{n-1}]$
40. Find area using right Riemann sums	$A = \Delta x\,[y_1 + y_2 + y_3 + \ldots + y_n]$
41. Find area using midpoint rectangles	$A = \Delta x\left[\frac{y_0+y_1}{2} + \frac{y_1+y_2}{2} + \ldots + \frac{y_{n-1}+y_n}{2}\right]$
42. Find area using trapezoids	$A = \frac{\Delta x}{2}\,[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n]$
43. Solve the differential equation ...	Separate Variables. Integrate both sides and solve for y.
44. Meaning of $\int_a^x f(t)\,dt$	The accumulation function: accumulated area under $f(x)$ starting at a constant a and ending at x.
45. Given a base, cross sections perpendicular to the x-axis are squares	$V = \int_a^b (\text{base})^2\,dx$ $[f(x) - g(x)]^2$
46. Find where the tangent line to $f(x)$ is horizontal	where $f'(x) = 0$
47. Find where the tangent line to $f(x)$ is vertical	where $f'(x) =$ undefined (set denominator $= 0$)
48. Find the minimum acceleration given $v(t)$	Find $a(t) \longrightarrow v'(t) = a(t)$ Set $a'(t) = 0$ or $a'(t) =$ undefined and make sign chart Determine where $a'(t)$ changes from $-$ to $+$.

49. Approximate the value of $f(0.1)$ by using the tangent line to f at $x = 0$	Find equation of tangent line at $x=0$. Plug 0.1 in for x and solve for y
50. Given the value of $F(a)$ and the fact that the anti-derivative of f is F, find $F(b)$	$\int_a^b f(x)\,dx = F(b) - F(a)$ Solve for $F(b)$ using calculator
51. Find the derivative of $f(g(x))$	$f'(g(x)) \cdot g'(x)$ (Chain Rule)
52. Given $\int_a^b f(x)\,dx$, find $\int_a^b [f(x)+k]\,dx$	$\int_a^b f(x)\,dx + \int_a^b k\,dx$
53. Given a picture of $f'(x)$, find where $f(x)$ is increasing	Determine where $f'(x)$ is positive
54. Given $v(t)$ and $s(0)$, find the greatest distance from the origin of a particle on $[a,b]$	Make sign chart for $v(t)$ to find turning points. Integrate $v(t)$ using $s(0)$ to find $s(t)$. Find s(all turning points) to find distance from start and origin
55. Given a water tank with g gallons initially being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$, find a) the amount of water in the tank at m minutes	$g + \int_{t_1}^{t_2} [F(t) - E(t)]\,dt$
56. b) the rate the water amount is changing at m	$\frac{d}{dt}\int_{t_1}^{m} [F(t) - E(t)]\,dt = F(m) - E(m)$
57. c) the time when the water is at a minimum	$F(m) - E(m) = 0$ (also test endpoints)
58. Given a chart of x and $f(x)$ on selected values between a and b, estimate $f'(c)$ where c is between a and b.	Use a value $k > c$ and a value $h < c$: $f'(c) \approx \frac{f(k) - f(h)}{k - h}$
59. Given $\frac{dy}{dx}$, draw a slope field	Plug given points into dy/dx and draw line segments with indicated slopes at each point
60. Find the area between curves $f(x), g(x)$ on $[a,b]$	$A = \int_a^b [f(x) - g(x)]\,dx$ $f(x) > g(x)$
61. Find the volume if the area between $f(x), g(x)$ is rotated about the x-axis	$V = \pi \int_a^b [R^2 - r^2]\,dx$
62. Find the interval that the velocity is increasing.	$v(t)$ is increasing when $v'(t) = a(t)$ is +. Set $v'(t) = 0$ or $v'(t)$ = undefined and make sign chart